Optimal Orientation of Symmetric Directional Antennas on a Line

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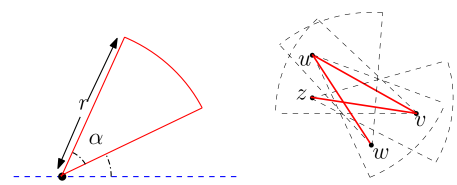
Abstract

In this paper, we study the problem of optimal orientation of directional antennas on a line in the symmetric model of communication. We propose an optimal algorithm to find the minimum radius and the orientation of antennas, when antennas are placed on a point set P on a line, and each antenna has angle less than π. We show that the connected graph induced by this optimal orientation is a 7-hop spanner with respect to the unit disk graph of P. Moreover, we present a deterministic local routing algorithm that is guaranteed to find a path between any pair of antennas in the communication graph whose number of edges is at most 7 times the number of edges between that pair in the unit disk graph.

1 Introduction

Wireless networks have received cosiderable attention in recent years due to their vast applications in various areas [11, 12]. Most of the time, wireless networks are modelled as a set P of n wireless nodes, where each node is equipped with an omni-directional antenna whose coverage area is a disk. Assuming identical transmission range for antennas, one can properly scale distances to make this transmission range equal to unit, and hence, the communication graph of antennas becomes equal to the *unit disk graph* of P, in which two antennas are connected if and only if the distance between them is at most unit.

Recent attention in the area of wireless networks has shifted from omni-directional antennas to *directional antennas*, due to their desirable properties such as improving security and reducing overlap [3]. A directional antenna can focus its transmission energy in a specific direction by narrowing coverage area, which is modelled by a sector of a fixed angle α and a radius r (see Figure 1(a) for an example). Antennas at different nodes can be oriented in different directions. There are two main models of communication in networks with directional antennas. In the *asymmetric* model, each antenna has a directed link to any other node that lies in its coverage area. In the *symmetric* model, there exists a link between two antennas u and v, if and only if u lies in the coverage area of v, and v lies in the coverage area of u. The symmetric model of communication is more practical, especially in networks where *handshaking* is required before transmitting data [7]. An example is illustrated in Figure 1(b).



(a) (b)

Figure 1: (a) A directional antenna. (b) A symmetric communication graph.

A network is called a *spanner*, if there is a path between any pairs of nodes, within a guaranteed ratio to the shortest paths between those nodes in an underlying base graph. This ratio is also called the *stretch factor* [14]. While the finite stretch factor is sufficient for existence of such a path between nodes through the network, the problem of efficiently finding the shortest path is central to many fields such as robotics and communication networks. In many cases, a node is not aware of the whole structure of the graph, and must learn this information through exploration. Algorithms for routing in these types of environments are called *local routing algorithms*. In local routing, for routing from a source point s to a destination point t, the current point u only knows about its neighbors and the location of t and should decide the next movement only using this information. A routing algorithm is *c-competitive* if the total distance traveled by the algorithm from any point s to any destination t, is not more than c times the length of the shortest path between those nodes in the graph. Parameter c is called the *competitive ratio* of the algorithm [4].

In this paper, we focus on the 1-dimensional version, where directional antennas are located on a set of points along a line. We assume the symmetric model for communication between the antennas. First we study the

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optimal orientation that, while it results in connectivity of the network, requires a minimum radius for the antennas. Then we prove that the resulting communication graph is a spanner with a constant stretch factor and also present a competitive local routing algorithm for this communication graph.

Related Work. The connectivity of communication graphs in the symmetric model was first studied by Ben-Moshe *et al.* [2]. They considered a limited setting (i.e., quadrant antennas and half-strip antennas) in which the orientation of antennas were chosen from a fixed set of directions. They showed how to orient antennas so that the communication graph becomes connected. Subsequent studies considered a more general setting, where each antenna can have an arbitrary orientation. Carmi *et al.* [7] proved that for $\alpha \geq \pi/3$, it is always possible to orient antennas so that the induced graph is connected. However, in their construction, the radius of the antennas are related to the diameter of the set of nodes, and hence the communication graphs can have a very large stretch factor, e.g., $O(n)$, compared to the original unit disk graph (i.e., the omni-directional graph of radius 1). Therefore, subsequent work considered the radius and stretch factor of the communication graph and proposed some approximation algorithms to minimize these factors. Aschner *et al.* [1] presented an algorithm to orient the antennas with angle $\pi/2$ and radius $14\sqrt{2}$ to obtain a 8-hop spanner, assuming that the unit disk graph of the nodes is connected. In a *t-hop spanner*, the number of hops (i.e., links) in a shortest link path between any pair of nodes is at most t times the number of hops in the shortest link path between those two nodes in the base graph, which happens to be a unit disk graph in this case. Tran *et al.* [15] improved the radius for the case $\alpha = \pi/2$ to 9. Dobrev *et al.* [10] showed that the connectivity problem is NP-hard for $\alpha < \pi/3$, where the radius is 1, and showed how to construct hop spanners for various values of $\alpha \geq \pi/2$.

Moreover, the problem of assigning transmission ranges to the omni-directional antennas placed arbitrarily on a line in order to achieve a strongly connected communication network with minimum total power consumption, was studied in the literature. Kirousis *et al.* [13] proposed an $O(n^4)$ time algorithm to obtain an optimal solution for this problem. Then, Das *et al.* [9] and Carmi *et al.* [6] improved the running time to $O(n^3)$ and $O(n^2)$, respectively. Also, Clementi *et al.* [8] considered the range assignment and stretch factor for noted problem. They presented a 2-approximation algorithm for the range assignment with running time $O(hn^3)$, where any pair of stations can communicate in at most h hops, to have a spanner with respect to the number of links. Furthermore, Carmi *et al.* [6] proposed a polynomial time algorithm to find the minimum radius whose induced communication graph becomes a t-spanner, for any $t \geq 1$. This problem was also studied for the asymmetric model of communication and Caragiannis *et al.* [5] proved that for a set of n points on a line, $0 \leq \alpha < \pi$ and $r > 0$, there exists an orientation of sectors of angle α and radius r at the points so that the communication graph is strongly connected if and only if the distance between points i and $i+2$ is at most r, for any $i = 1, 2, \ldots, n-2$.

Our Results. In this paper, we study the problem of orienting a set of directional antennas on a line, to make the resulting communication graph connected, while the transmission range (radius) is minimized. We present an efficient algorithm that finds an orientation with optimal radius in linear time. This is indeed the first algorithm for the problem that achieves an optimal radius.

We prove that the communication graph obtained from this orientation is a 7-hop spanner, meaning that the shortest link distance between any pair of nodes in the resulting communication graph is at most 7 times the shortest link distance between those nodes in the unit disk graph of the points. In other words, we compare the stretch factor of our connected directional network to that of a connected omni-directional network. We also present an algorithm to route locally in this communication graph with a competitive ratio of 7. To the best of our knowledge, there is no previous result for routing locally and competitively in the communication graph of directional antennas, and hence, we are presenting the first such result in this paper.

2 Preliminaries

Let P be a set of points in the plane, and G be a graph on the vertex set P. For two points $p, q \in P$, we denote by $\delta_G(p, q)$ the *shortest link distance* between p and q in G, i.e. the minimum number of edges needed to connect p and q in G. If the graph G is clear from the context, we simply write $\delta(p, q)$ instead of $\delta_G(p, q)$. A path that realizes $\delta(p, q)$ is called a *shortest path*. For two points p and q in the plane, the Euclidean distance between p and q is denoted by $\|pq\|$. Throughout the paper, the farthest and nearest neighbors are in terms of the Euclidean distance.

For a point set P, we denote by $UDG(P)$ the *unit disk graph* of P, i.e. a graph on the vertex set P in which two vertices are connected if and only if they are within distance unit of each other. Throughout this paper, we assume that $UDG(P)$ is connected, which is necessary for the omni-directional network on P to be connected. We also assume that the largest edge in $UDG(P)$ has *unit* length. This assumption can be easily realized by a proper scaling of the point set.

Given two graphs H and G on the vertex set P, we call H a *t-hop spanner* with respect to G, if for any two vertices u and v in G, we have $\delta_H(u, v) \leq t \cdot \delta_G(u, v)$.

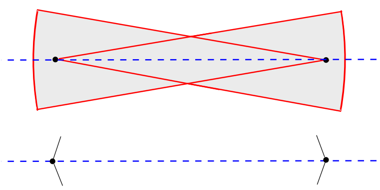

Figure 2: The right and left orientations.

Given a routing algorithm A on G, we say that A is *c-competitive*, if for any pair of vertices s and t, the number edges on the path found by A from s to t in G is at most c times the shortest link distance between s and t in $UDG(P)$.

3 An Algorithm for the Optimal Orientation

In this section, we propose a linear-time algorithm for the optimal orientation in one dimension. More precisely, we direct antennas located on a point set P placed on a line, to obtain a connected communication graph, while minimizing the radius. The challenging part of the problem is when $\alpha < \pi$. (The case $\alpha \geq \pi$ is pretty straight-forward.) In this case, each antenna covers at most a half-plane. Since antennas are located on a line, their orientation can be viewed as either left or right, ignoring the value of α. We denote antennas facing left and right using symbols $\rangle$ and $\langle$, respectively (see Figure 2).

We first present a lemma, describing a useful property of the optimal orientations.

Lemma 1 *There is always an optimal orientation for the antenna set P, in which no three consecutive antennas are in the same direction.*

Proof. Given an optimal orientation, let a *bad triple* be a set of three consecutive antennas with the same orientation. We observe that for any bad triple, the middle antenna is not needed for the connectivity of its left and right neighbors, so we can change its direction without harming the connectivity of the two neighboring antennas. Since the left and right antennas remain connected regardless of the direction of the middle antenna, the middle antenna also remains connected in either direction. Now, consider the optimal solution with the minimum number of bad triples. If this number is not zero, then we can decrease it by the above observation (finding a bad triple and changing the direction of the middle one). Therefore, there is always an optimal solution with no bad triples. □

Using Lemma 1, we can devise a dynamic programming approach to find an optimal orientation in linear time. In an orientation of antennas, let a *block* be a maximal sequence of consecutive antennas starting with one or more antennas facing to the right, and followed with one or more antennas facing to the left. For example, the orientation $\langle\rangle\langle\langle\rangle\langle\langle\langle\rangle\rangle$ consists of three blocks of lengths 2, 3, and 5, respectively. In an optimal orientation, the leftmost (resp., rightmost) antenna is directed to the right (resp., left), and hence, an optimal orientation can be viewed as a series of blocks. By Lemma 1, there is an optimal orientation in which all blocks are either $\langle\rangle$, $\langle\langle\rangle\rangle$, $\langle\langle\rangle$, or $\langle\rangle\rangle$. We try to find such an optimal orientation using dynamic programming.

We observe that the following two conditions are necessary and sufficient for an orientation to have a connected communication graph:

(I) The subgraph of each block is connected.

(II) Each block has edges to its neighboring blocks.

These conditions guarantee that the graph is composed of a set of connected components, each of which connected to its two neighboring components, and hence, the whole graph is connected. By the first condition, the nodes in a block must be able to communicate without getting help from other blocks. Since the minimum radius for block $\langle\langle\rangle\rangle$ is equal to the maximum of the radii for two blocks $\langle\langle\rangle$ and $\langle\rangle\rangle$, we only need to consider these three types: $\langle\rangle$, $\langle\langle\rangle$, and $\langle\rangle\rangle$. This condition holds if and only if the leftmost and rightmost antennas in the block are connected to at least one other node. It means that the leftmost $\rangle$ and $\langle$ must be connected, and analogously the rightmost ones should cover each other. This suggests the lower bound on the radius in this orientation.

By the second condition, two neighboring blocks should be able to directly communicate. Two consecutive blocks $\mathcal{B}_1$ and $\mathcal{B}_2$ ($\mathcal{B}_1$ is to the left of $\mathcal{B}_2$) are connected to each other, if and only if the rightmost $\langle$ in $\mathcal{B}_1$ is connected to the leftmost $\rangle$ in $\mathcal{B}_2$. So the distance between these two nodes is another lower bound on the radius.

By the structure of the blocks, there is always an optimal orientation, which ends with the patterns illustrated in Figure 3 (since the rightmost part of the configuration is considered, the block $\langle\rangle\rangle$ is always as good as the block $\langle\langle\rangle$ as illustrated in case 1). As we can see in Figure 3, the $\langle\rangle$ setting appears in every case. Now let $x_1 < x_2 < \cdots < x_n$ be the position of the antennas P on the real line, we define r_i to be the optimal radius for the subproblem restricted to the first i antennas with an extra restriction that the last block has the $\langle\rangle$ setting (like cases 2 and 3). Thus, we have the following recursive formula for r_i, when $i > 4$:

$$r_i = \min\{\max\{r_{i-2}, x_i - x_{i-3}\}, \max\{r_{i-3}, x_i - x_{i-4}\}\}$$

Actually, in the subproblems like cases 2 and 3, we need radius at least $x_i - x_{i-3}$ and $x_i - x_{i-4}$, respectively for the connectivity condition (II) to hold for the

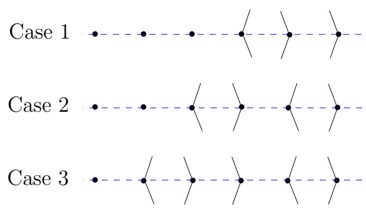

Figure 3: Optimal substructures for orienting antennas.

last two blocks (these radii surely guarantee that condition (I) holds for these blocks). Moreover, by the observation in Figure 3, to have connectivity condition (I) for the last block in case 1, the radius must be at least $x_n - x_{n-2}$. So, the optimal radius is equal to $\min\{r_n, \max\{r_{n-1}, x_n - x_{n-2}\}\}$. The following pseudocode shows the dynamic programming algorithm based on the above recursive formula.

Algorithm 1 OPTIMAL ORIENTATION

input: $x_1, x_2, \cdots, x_n$ the position of antenna set P
output: Optimal radius r

1: **for** $i \leftarrow 1$ to 4 **do**
2: $\quad r_i \leftarrow x_i - x_1$
3: **for** $i \leftarrow 5$ to n **do**
4: $\quad r_i \leftarrow \min\{\max\{r_{i-2}, x_i - x_{i-3}\}, \max\{r_{i-3}, x_i - x_{i-4}\}\}$
5: $r \leftarrow \min\{r_n, \max\{r_{n-1}, x_n - x_{n-2}\}\}$

Algorithm 1 only computes the optimal radius. However, it can be easily modified to output the optimal orientation as well, by storing in a second table the directions minimizing the radii in lines 4 and 5 of the algorithm. All together, we get the following result.

Theorem 2 *Let P be a set of points on a line. There exists a linear-time algorithm that finds an optimal radius r and an optimal orientation of antennas with angle $\alpha < \pi$ and radius r located on P, such that the resulting communication graph $\mathcal{G}(P)$ is connected.*

Remark. Given that a linear-time algorithm exists for the optimal orientation in one dimension, one may be tempted to find a simpler greedy strategy for the problem. For example, for the decision version of the problem which asks for a fixed radius r, if an orientation exists that makes the resulting communication graph connected, the following greedy strategy seems promising: starting from the leftmost antenna p, find the rightmost antenna q which is within distance r of p. We then orient p to the right and q to the left. All other antennas between p and q can be safely oriented to the right. We then repeat this process, with the antenna to the left of q as p. It is not hard to see that this greedy strategy may not work properly (see Figure 4).

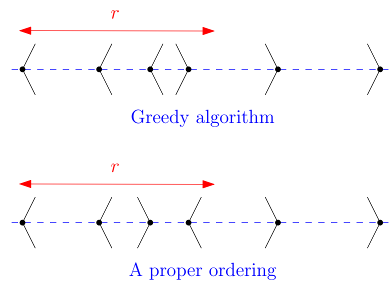

Figure 4: The greedy algorithm fails to build a connected communication graph using radius r.

4 Stretch Factor of the Optimal Orientation

In this section, we prove that the communication graph obtained by Algorithm 1 is a t-hop spanner with respect to the unit disk graph of P.

Theorem 3 *Let P be a point set on a line such that $UDG(P)$ is connected. The communication graph $\mathcal{G}(P)$ obtained by the optimal orientation in Algorithm 1 is a 7-hop spanner of $UDG(P)$.*

Proof. Consider an arbitrary edge $(u, v) \in UDG(P)$. We show that $\delta(u, v)$ in $\mathcal{G}(P)$ is at most 7, while all possible orientations of the antennas located on u and v are considered. Assume w.l.o.g. that u is to the left of v. There are three possible cases.

- Antennas at u and v have right and left directions, respectively ($\langle\rangle$): r is greater than or equal to the unit to guarantee the connectivity of the communication graph. So, there is a direct edge between u and v in $\mathcal{G}(P)$.

- Antennas at u and v have the same directions (either $\langle\langle$ or $\rangle\rangle$): We assume w.l.o.g. that these antennas have $\langle\langle$ setting. Let v' be the nearest neighbor of v in $\mathcal{G}(P)$ (v and v' are in the same block). If u and v' are in the same block, there is a direct edge between them and so $\delta(u, v) = 2$. Otherwise, consider the block $\mathcal{B}$ that is located to the left of the block of v. According to the connectivity condition (II), v' connects to a point in block $\mathcal{B}$, such that this point has a neighbor u' in this block with left orientation. Since u' lies between u and v, the distance between u and u' is less than or equal to unit and thus, by the previous case, u connects to point u'. Therefore, $\delta(u, v)$ is at most 4 in this case.

- Antennas at u and v have left and right directions, respectively (i.e., $\rangle\langle$): Consider the nearest neighbors of u and v, and call them u' and v', respectively. u and u' are in a block $\mathcal{B}_1$, and v and v' are in a block $\mathcal{B}_2$. If $\mathcal{B}_1$ and $\mathcal{B}_2$ are two consecutive blocks, due to the connectivity condition (II), u'

and v' connect to each other with a direct edge, and hence $\delta(u,v) = 3$. Otherwise, u' connects directly to a point u'' in its right block, and v' connects directly to a point v'' in its left block. If u'' and v'' are in a common block, there is an edge between them and $\delta(u,v) = 5$. But if they are in two different blocks, we need three edges to connect them to each other. Since at first u'' and v'' must connect to their nearest neighbors, who have right and left directions respectively, we can then use the first case of the proof to connect these neighbors with one more edge. So, $\delta(u,v)$ is at most 7 in this case.

Now, let p and q be two arbitrary points in P, and $p_0 = p, p_1, \ldots, p_t = q$ be the shortest link distance between p and q in $UDG(P)$. Since $\|p_i p_{i+1}\|$ is less than or equal to the unit, each link (p_i, p_{i+1}) either exist or is replaced by a path of link length at most 7 in $\mathcal{G}(P)$. Therefore, the communication graph $\mathcal{G}(P)$ is a 7-hop spanner. □

5 Local Routing for the Optimal Orientation

In the previous section, we proved that to transfer data between two points that communicate with each other directly in the unit disk graph, there is a path with at most 7 hops in the resulting communication graph of optimal orientation. Although we proved the existence of such path, we need to provide a routing algorithm to find it. Here, we propose a local routing algorithm for communication graph $\mathcal{G}(P)$ of the optimal orientation of the antenna set P.

According to the orientation of antennas (left or right) in the communication graph $\mathcal{G}(P)$, each point connects to some points located either to its left or its right. Therefore, the direction of transfer is predetermined and in each state we just need to choose the best neighbor of the current point for the next step. We assume that the neighbors of each point are sorted in their x-coordinates. We propose Algorithm 2 to route from s to t in graph $\mathcal{G}(P)$. During the algorithm, if the orientation of the antenna on the current point u is in the direction of the destination, we go to the farthest neighbor of u in order to close the gap to t as much as possible, and if the orientation of the antenna located on u is in opposite of the direction of the destination, we go to the nearest neighbor in order to increase the distance to t the least.

To prove the correctness of the algorithm, we assume w.l.o.g that s is to the left of t, and then show that we will certainly reach from s to t after visiting a finite number of points. We denote by $\pi(s,t)$ the path obtained by Algorithm 2. Moreover, we define the *head* of a block to be the rightmost antenna with right direction in that block.

Lemma 4 *In $\pi(s,t)$, each antenna with right direction, except s and t, is the head of a block, and these heads appear in the ascending order of their x-coordinates along $\pi(s,t)$.*

Algorithm 2 ROUTING($\mathcal{G}(P), s, t$)

input: Communication graph $\mathcal{G}(P)$, point s and t
output: Routing from s to t

```
while s is not directly connected to t do
    if the antenna on s is oriented toward t then
        u ← farthest neighbor of s
        ROUTING(G(P), u, t)
    else
        u ← nearest neighbor of s
        ROUTING(G(P), u, t)
```

Proof. Every antenna with left direction in $\pi(s,t)$, except t, can not see t. Therefore, we go to its nearest neighbor, which has right direction and is therefore the head of a block. In Algorithm 2, if the current point u is a head, we go toward t or to the farthest neighbor of it, say u'. Since the direction of a head is right, by the connectivity condition (II), u' is located in a block which lies to the right of u. Now, either u' directly connects to t, or we go to the head of its block, whose x-coordinate is greater than u. □

By Lemma 4, the points in $\pi(s,t)$ are alternating heads of blocks in ascending x-coordinates. Since the number of blocks is finite, the proposed routing algorithm reaches from s to t after a finite number of steps by the invariant property. (If it passes over t, after one backward movement it certainly gets to t.)

5.1 Competitive Ratio of the Routing Algorithm

Here, we compare the path $\pi(s,t)$, obtained by Algorithm 2 on $\mathcal{G}(P)$, with a shortest path between s and t in $UDG(P)$ and show that Algorithm 2 can route locally and competitively on graph $\mathcal{G}(P)$. So, we first prove a lemma.

Lemma 5 *If h_1, h_2, h_3, and h_4 are four consecutive heads in $\pi(s,t)$, then $\|h_1 h_4\| \geq r$.*

Proof. If $\|h_1 h_4\| < r$, there is an antenna p in the block to which h_3 belongs, such that the direction of p is left and its Euclidean distance to h_1 is less than r. Thus, there is a direct edge between h_1 and p. Since h_1, h_2 and h_3 are consecutive heads in $\pi(s,t)$, in the routing algorithm we go along the path from h_1 to an antenna q with left direction, which is located between h_2 and h_3, and then go from q to h_2 with a movement. We know that $\|h_1 q\| < \|h_1 p\|$, and that both p and q are neighbors of h_1. (The status of antennas can be illustrated as $\langle^{h_1} \cdots \langle^{h_2} \;\rangle^{q} \cdots \langle^{h_3} \;\rangle^{p} \cdots \langle^{h_4}$.) Therefore, in the routing algorithm, we go after h_1 to its farthest neighbor which is not q. But, this contradicts the assumption that h_1 and h_2 are consecutive heads along the path, and this completes the proof. □

Corollary 1 *Since the antennas in $\pi(s,t)$ have alternating left and right directions, we use at most six edges to move from h_1 to h_4, and after these steps, h_4 becomes at least r times closer to t than h_1, i.e., $\|h_1t\| - \|h_4t\| \geq r$.*

If the distance between two arbitrary points s and t is in the range $[(k-1)r, kr]$ for a positive integer k, by Corollary 1, after $6(k-1)$ steps, the Euclidean distance between the current point u and t becomes less than or equal to r. On the other hand, we proved in Section 4 that for any two points u and v in $\mathcal{G}(P)$ with distance less than or equal to unit, $\delta(u,v) \leq 7$. We can easily generalize this result to the case when the distance between two points is at most r. Therefore, for the current point u and the destination point t, there is a path with at most 7 edges connecting them, which is exactly the path found by Algorithm 2. Therefore, for reaching from s to t, we pass at most $6(k-1)+7 = 6k+1$ edges, and hence, $|\pi(s,t)| \leq 6k+1$.

In $UDG(P)$, by passing each edge in a shortest path from s to t, we get closer to t by at most one unit. So, if the distance between two arbitrary points s and t is in the range $[(k-1)r, kr]$, we have $\delta_{UDG}(s,t) \geq kr$, and because r is greater than or equal to unit, we have $\frac{|\pi(s,t)|}{\delta_{UDG}(s,t)} \leq (6 + \frac{1}{k})$. The following theorem summarizes the result.

Theorem 6 *Let P be a set of points on a line such that $UDG(P)$ is connected. Algorithm 2 is a 7-competitive routing algorithm with respect to the $UDG(P)$, for the communication graph $\mathcal{G}(P)$ computed by Algorithm 1.*

6 Conclusion

In this paper, we studied the problem of orienting directional antennas in the symmetric model of communication, and presented an efficient linear-time dynamic programming algorithm for finding an optimal orientation with a minimum radius in one dimension. Moreover, we showed that the induced communication graph of the optimal orientation is a t-hop spanner, for a small stretch factor $t \leq 7$. We also presented a 7-competitive local routing algorithm on the resulting graph.

Several interesting problems remain open. The main question is how to extend the results of this paper to two and higher dimensions. In particular, there is a 2-approximation algorithm for the problem (in a limited setting) in two dimensions. However, it is not yet known whether the problem in the plane is NP-hard, or can be solved optimally in polynomial time. Moreover, finding routing algorithms for networks with directional antennas in two and higher dimensions remains open.

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