Burning the Medial Axis

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Figure 1: The burn time of a 2d shape (top) and the same shape with a perturbed boundary (bottom).

Initially proposed by Blum in 1967, the medial axis of a shape consists of the union of all centers of maximally inscribed balls. The medial axis is one of the most commonly used tools for understanding shape, as it is homotopy equivalent to the original object, has codimension one, and is centrally located. In addition, it is used as a component in building skeletons that are of smaller dimension than the original object, but which capture the shape in a more compact but still useful representation. However, the medial axis is unstable to perturbations; even small changes in the boundary of the shape result in large changes in the medial axis.

Methods for pruning the medial axis are usually guided by some measure of significance, with considerable work done for both 2 and 3 dimensional shapes. However, the majority of significance measures over the medial axis are locally defined and hence unable to capture more global features, or are difficult to compute and sensitive to perturbations on the boundary. In general, there are no skeletons which provably capture the correct topology, are central to the object, are always result in a curve skeleton for a 3-dimensional input.

In this talk, I will present recent work done in 2d and 3d to compute new significance measures on the medial axis. In 2d, the extended distance function (EDF), also called the burn time, was recently developed by Liu et [3], as well as related measures such as erosion

Figure 2: A figure of a dolphin, with the medial axis, erosion thickness, and resulting skeleton.

thickness and weighted EDF [1]. See Figure 1 for an illustration of this function. The EDF function was later generalized to the burn time function for 3 dimensional shapes, yielding both a mathematical framework for quantifying shape as well as an algorithm for approximating this function for a union of balls, which are commonly used for surface reconstruction and approximation [4]. In 3d, this also allows us to develop a definition of topologically accurate 1-dimensional skeletons; see Figure 2. These measures give practical methods for differentiating boundary noise from primary features, and can be used for shape alignment and recognition. In addition, there is both practical and theoretical evidence that these measures are robust under certain types of noise in the boundary [2], unlike the medial axis itself.

References

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